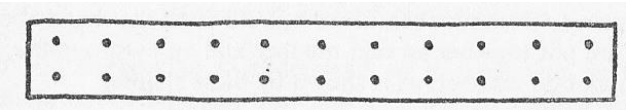
Sawyer's Vision in Elementary Mathematics

About a year ago I met up with an old friend for a beer and a chat. He'd been interested in my new career: I'd recently given up my day job as a business software developer to work from home writing and hopefully selling maths software to schools. He had worked as a maths teacher in schools some years ago and, as we were leaving, he passed me a paper bag and said he thought I might find these useful. Inside were four very old Pelican paperbacks all written by W. W. Sawyer. The top one, *Vision in Elementary Mathematics* was priced at five shillings!

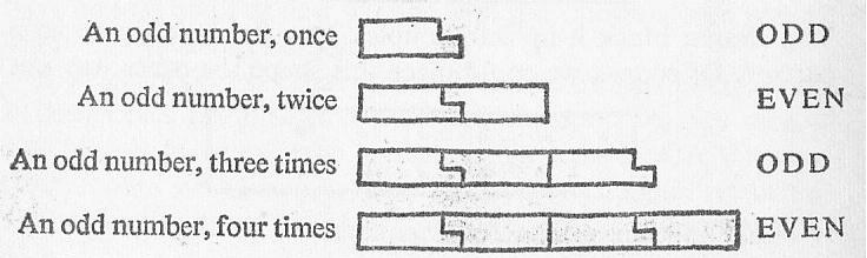
I'd been working on my project for a few years in my spare time so I already had quite a collection of activities and had gathered quite a lot of positive feedback from teachers already using it. Perhaps my guiding principle had been to try to use computers for what they do best: to show clear visual images and then to allow users to interact with those images and witness their effects: dragging the corners of a triangle, say, to change its shape or clicking on a decimal ruler to expand a small section into a whole new ruler. It seemed to me that too much maths software either just put pages on screens or offered engaging cartoon characters that cavorted about for a few minutes with voices and music before displaying the inevitable, dreary sum with its multiple choice answer.

Imagine then my delight on opening Sawyer to find someone who explained mathematics through simple, clear and sometimes gently amusing pictures.
In chapter one he presents his images for odd and even numbers which suddenly make it obvious why any two odd numbers must sum to an even number, three odds to an odd and so on.



An even number

An odd number



In chapter three we are already preparing for algebra. He states:

'It is a defect of most algebra books that they begin by developing a lot of machinery, and it is a long time before the learner sees what he can do with all this machinery. For example, he may learn to simplify 5(x + 3) - 4(2 - x) without seeing in just what circumstances he would feel a need to perform this calculation.'

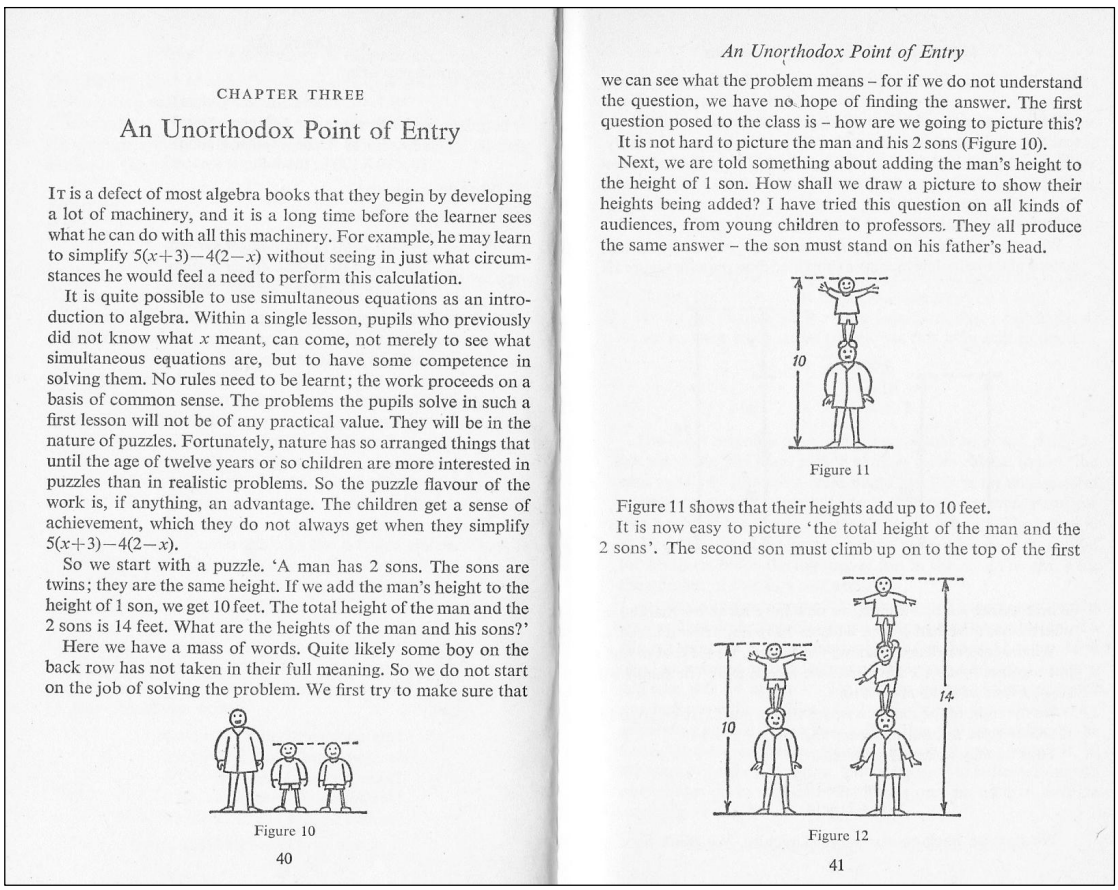
which brings me to Sawyer's language, which is no less clear and no less amusing than his drawings. Starting with a standard 1950s style wordy problem like this:

> Problem: A man has 2 sons. The sons are twins; they are the same height. If we add the man's height to the height of 1 son, we get 10 feet. The total height of the man and 2 sons is 14 feet. What are the heights of the man and his sons?

I can imagine for many adults just reading this problem might bring back some unpleasant feelings from maths lessons in their own secondary school. But Sawyer comes to the rescue:

'Here we have a mass of words. Quite likely some boy on the back row has not taken in their full meaning…'

Does he mean me? And then he goes on to visualise the situation. Just take a look at these two pages.



CHAPTER THREE

An Unorthodox Point of Entry

IT is a defect of most algebra books that they begin by developing a lot of machinery, and it is a long time before the learner sees what he can do with all this machinery. For example, he may learn to simplify $5(x+3)-4(2-x)$ without seeing in just what circumstances he would feel a need to perform this calculation.

It is quite possible to use simultaneous equations as an introduction to algebra. Within a single lesson, pupils who previously did not know what x meant, can come, not merely to see what simultaneous equations are, but to have some competence in solving them. No rules need to be learnt; the work proceeds on a basis of common sense. The problems the pupils solve in such a first lesson will not be of any practical value. They will be in the nature of puzzles. Fortunately, nature has so arranged things that until the age of twelve years or so children are more interested in puzzles than in realistic problems. So the puzzle flavour of the work is, if anything, an advantage. The children get a sense of achievement, which they do not always get when they simplify $5(x+3)-4(2-x)$.

So we start with a puzzle. 'A man has 2 sons. The sons are twins; they are the same height. If we add the man's height to the height of 1 son, we get 10 feet. The total height of the man and the 2 sons is 14 feet. What are the heights of the man and his sons?'

Here we have a mass of words. Quite likely some boy on the back row has not taken in their full meaning. So we do not start on the job of solving the problem. We first try to make sure that

Figure 10

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An Unorthodox Point of Entry

we can see what the problem means – for if we do not understand the question, we have no hope of finding the answer. The first question posed to the class is – how are we going to picture this?

It is not hard to picture the man and his 2 sons (Figure 10).

Next, we are told something about adding the man's height to the height of 1 son. How shall we draw a picture to show their heights being added? I have tried this question on all kinds of audiences, from young children to professors. They all produce the same answer – the son must stand on his father's head.

Figure 11

Figure 11 shows that their heights add up to 10 feet.

It is now easy to picture 'the total height of the man and the 2 sons'. The second son must climb up on to the top of the first

Figure 12

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Suddenly a boring maths problem is turning into a bit of fun with a father and his acrobatic sons. How much better is this than just being presented with the algebraic equivalent: m + s = 10

m + 2s = 14

Solve for m and s.

This gentle humour of the imagination is extended further when a new problem involves the subtraction of the son's height but I'll leave it there for now.

I opted to use chapter four: tricks, bags and machines as the basis for my first activity based on Sawyer's book.

He starts with the old trick: think of a number, add 3, double it and so on.
Again he encourages us to visualises the problem. There are a number of stones in a bag – we don't know how many but they represent the number thought of. We draw a bag therefore to represent an unknown number.
Add 3 – we can now draw three stones alongside the bag.
Double it – we draw another set. And so on.
Already Sawyer has given us a way in to the problem which, as with the man and his acrobatic sons, can be written in shorthand as algebra but only *after* we've visualised the problem.

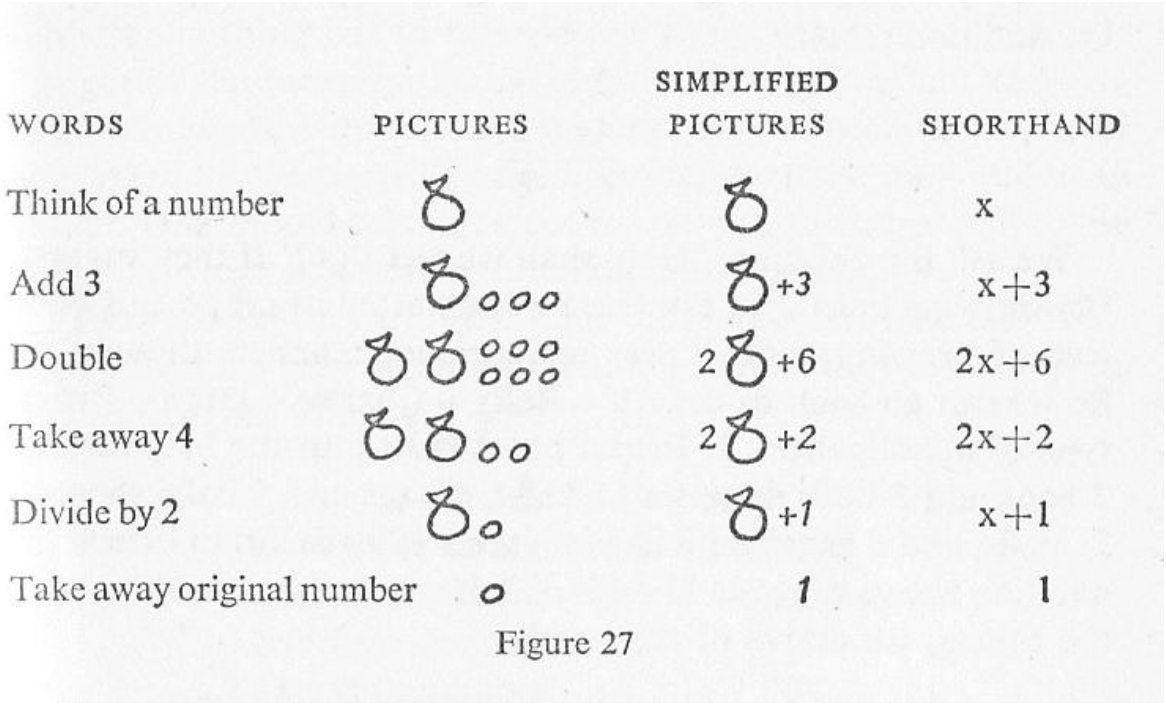

Figure 27

I googled for 'Sawyer' in an attempt to find his details so that I could apply for permission to create an activity based on his ideas. He was born in 1911 but I hoped he was still alive and felt sure he would be excited by the enormous potential we now have using computers and interactive whiteboards to help learners visualise their mathematics. The search led me to Mark Alder who had known Sawyer and had placed a number of Sawyer's articles on his website. He informed me that sadly Sawyer had recently died in Canada.

But I have his books and will continue to be inspired by his writings.

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